

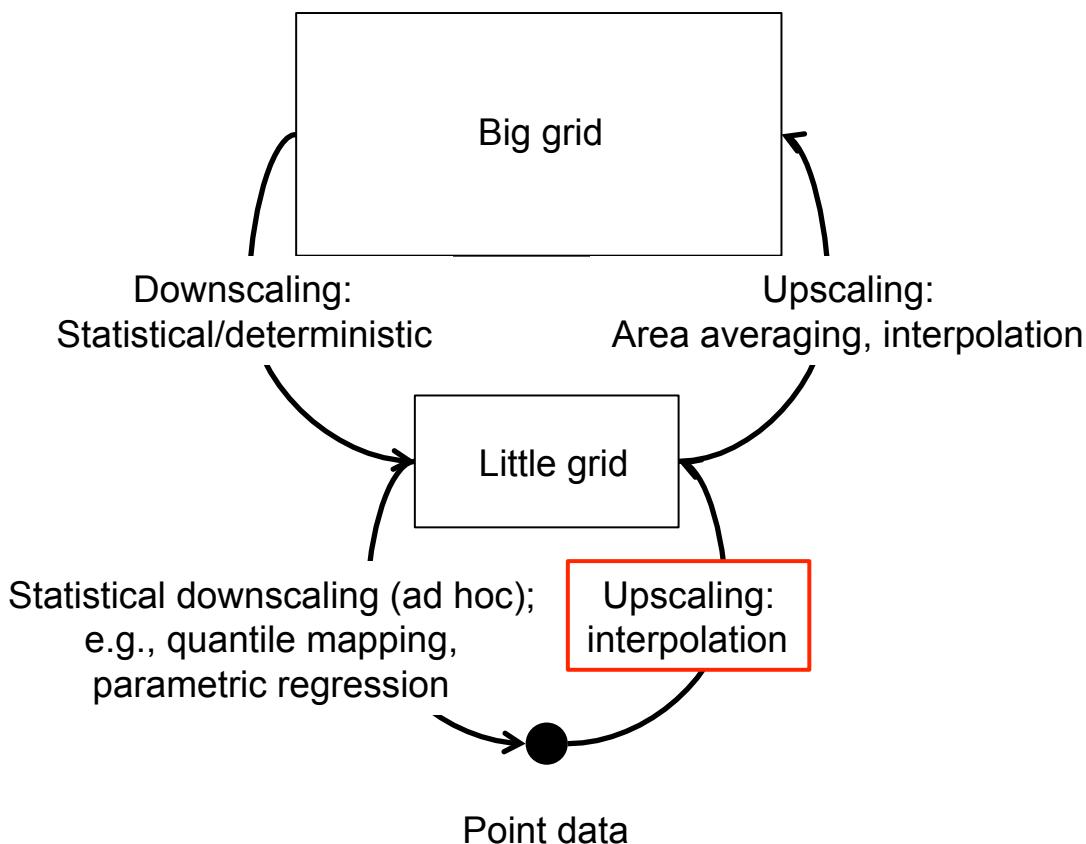
# Multi-scale variability and trends in observed surface air temperature: *Lessons from the intersection of statistical mechanics and climatology*

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Samuel S.P. Shen  
San Diego State University

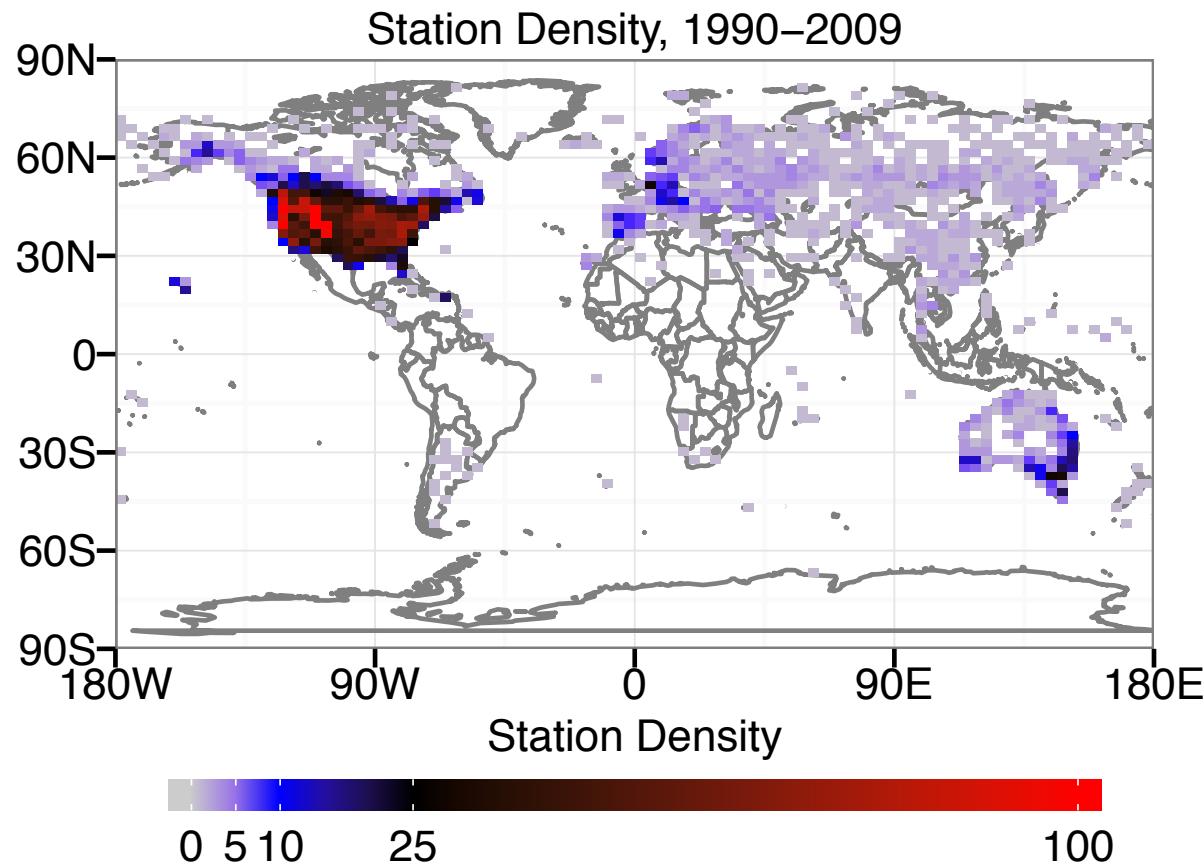


# Variability & scaling – a quick overview



- Global Historical Climatology Network overview
  - Probabilistic variability & moments
- I'll focus more on:
- Quantifying variability at point scale in space & relating it to larger scales
- I'll focus less on:
- The implications for studying risk (i.e., actual probabilities) & weather with climate change / decadal variability

# Global Historical Climatology Network – Daily (GHCND)

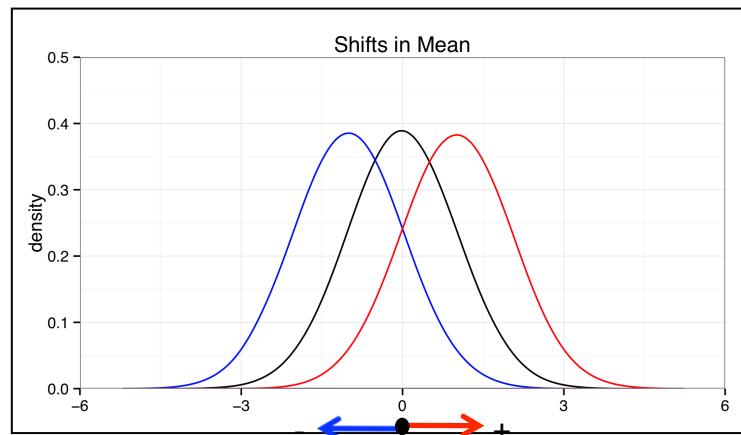
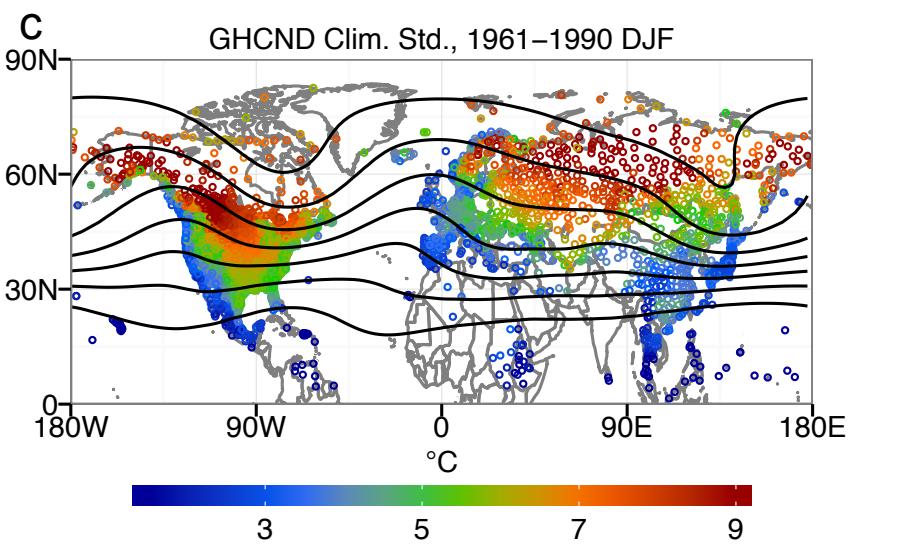
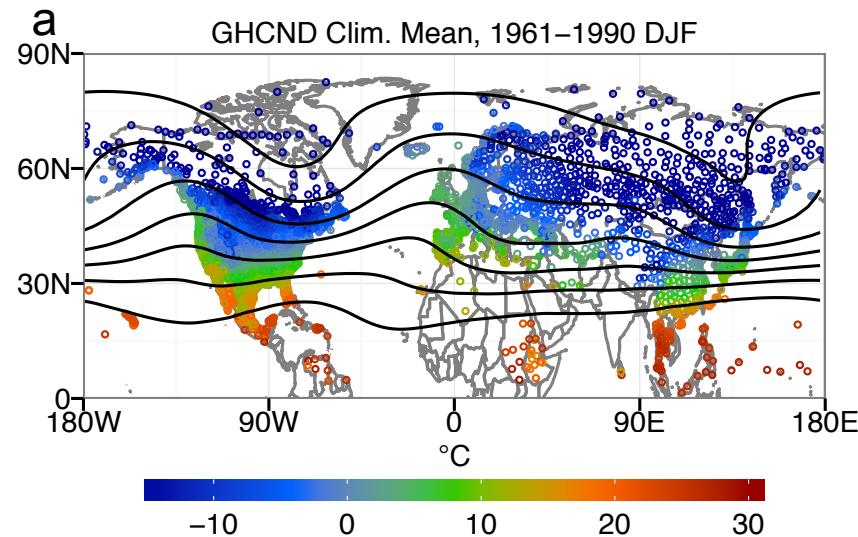


- 90,000+ weather stations globally, mostly precipitation records
- We study about 8,000 temperature anomaly records which pass QC

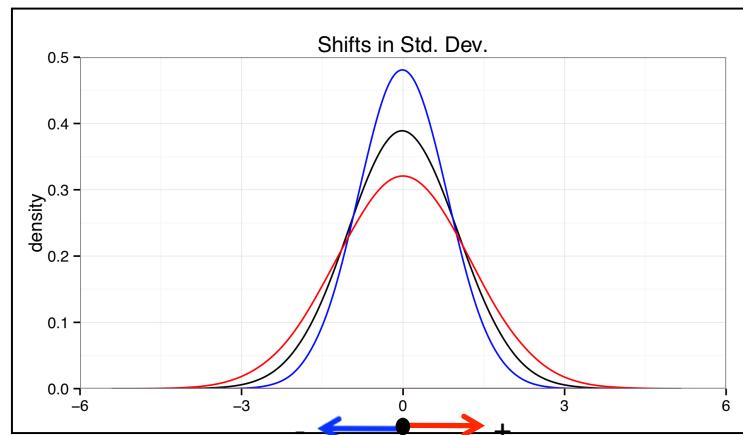
# Gaussian Moment Statistics

$$\mu_n = E[(X - \bar{X})^n]$$

-- 250mb height



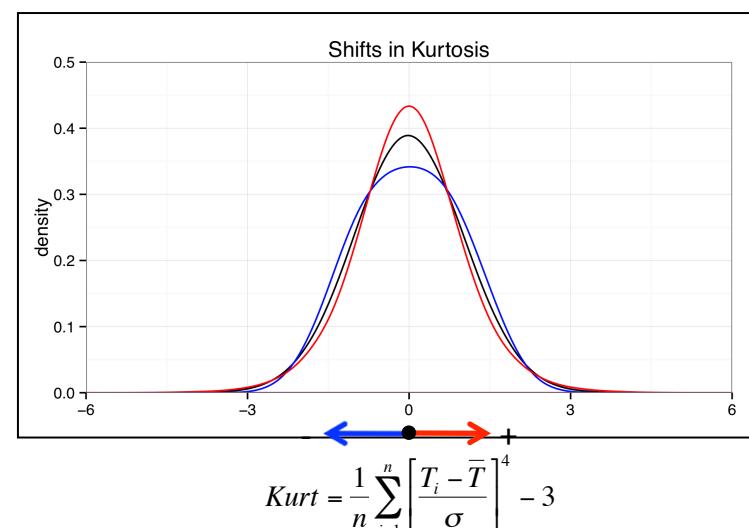
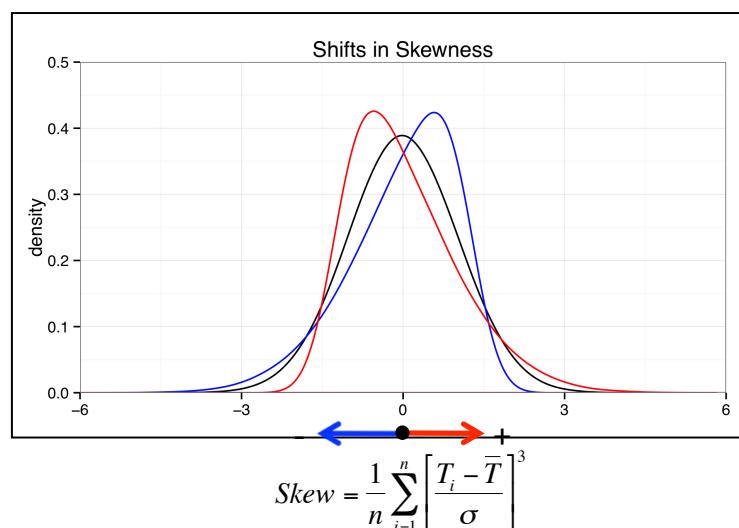
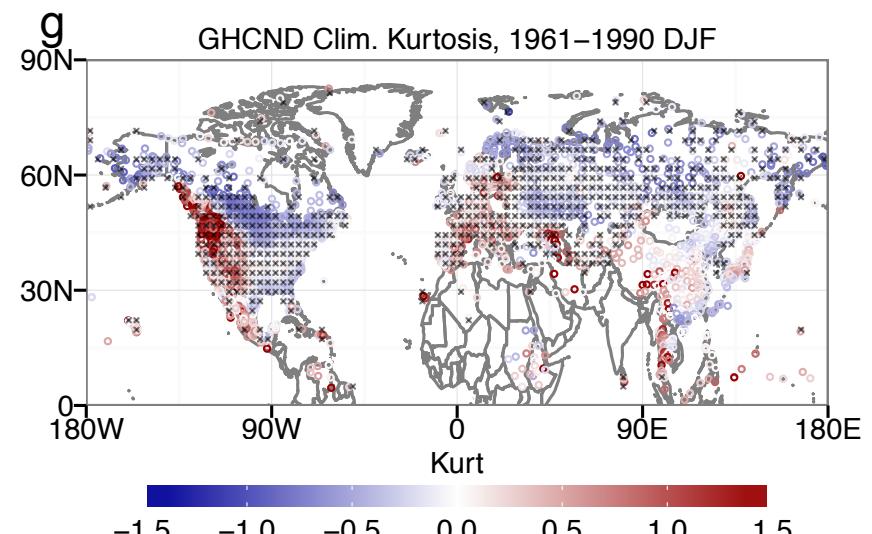
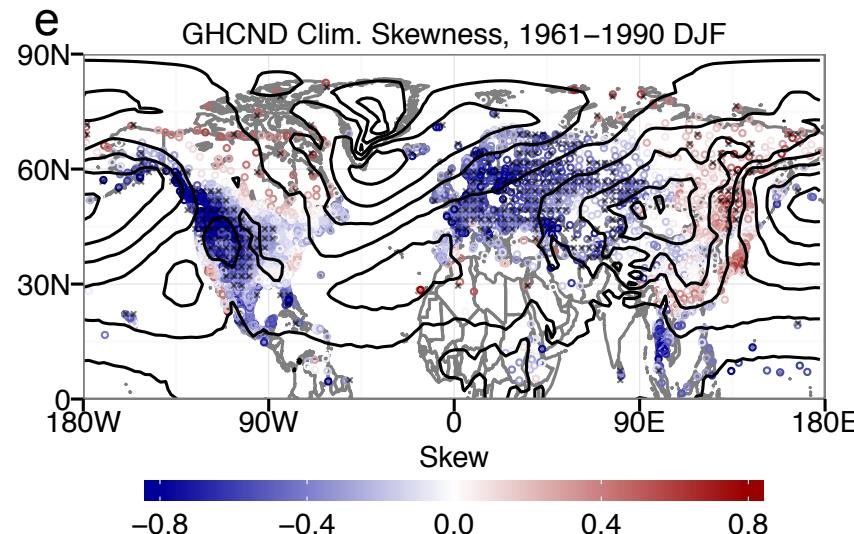
$$\bar{T} = \frac{1}{n} \sum_{i=1}^n T_i$$



$$\sigma = \left( \frac{1}{n} \sum_{i=1}^n [T_i - \bar{T}]^2 \right)^{\frac{1}{2}}$$

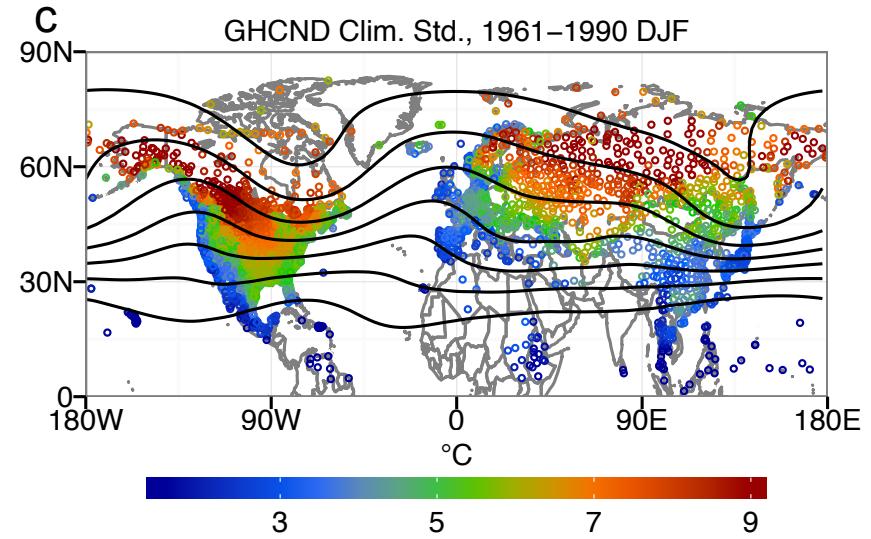
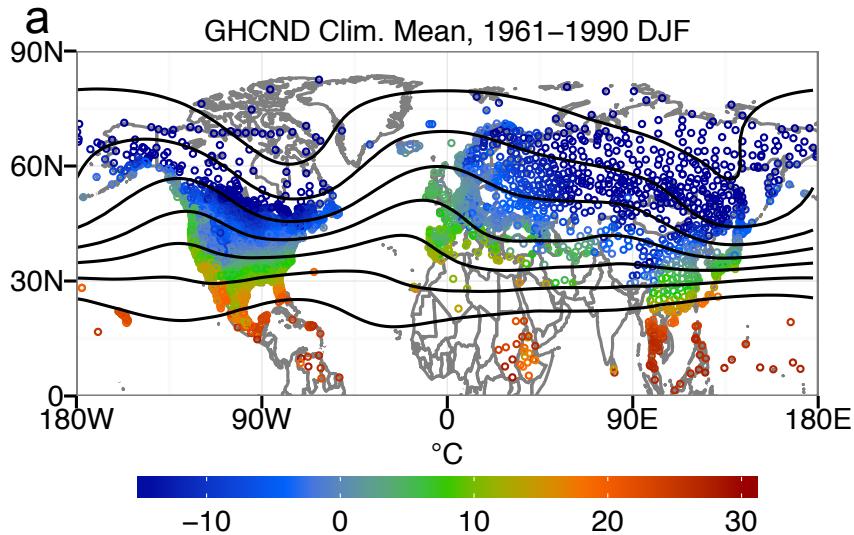
# Non-Gaussian Higher-Order Moments

-- SLP



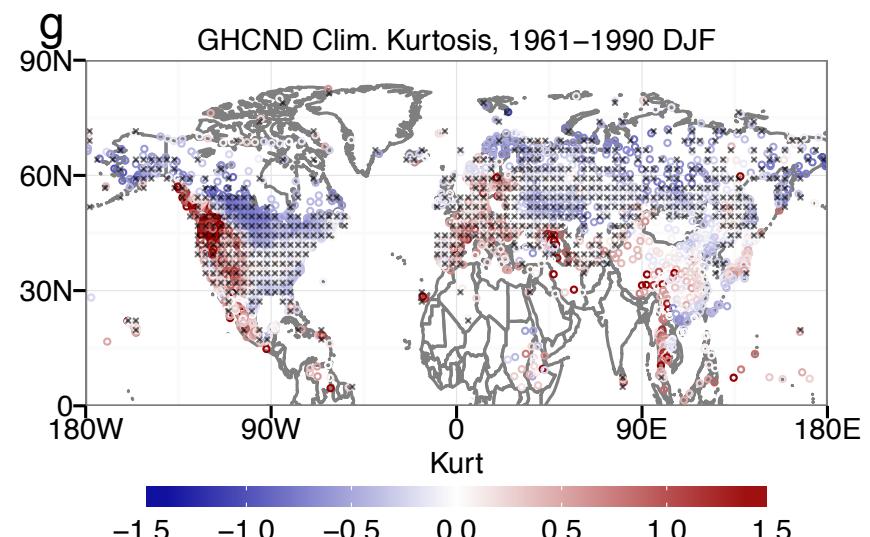
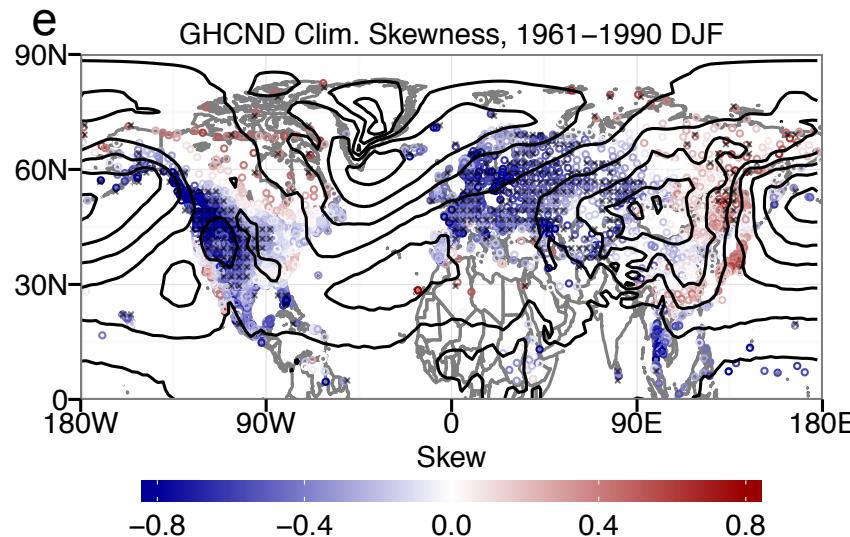
\*simulated from a skew-normal  
Note: non-zero fourth moments

\*simulated from t-distributions

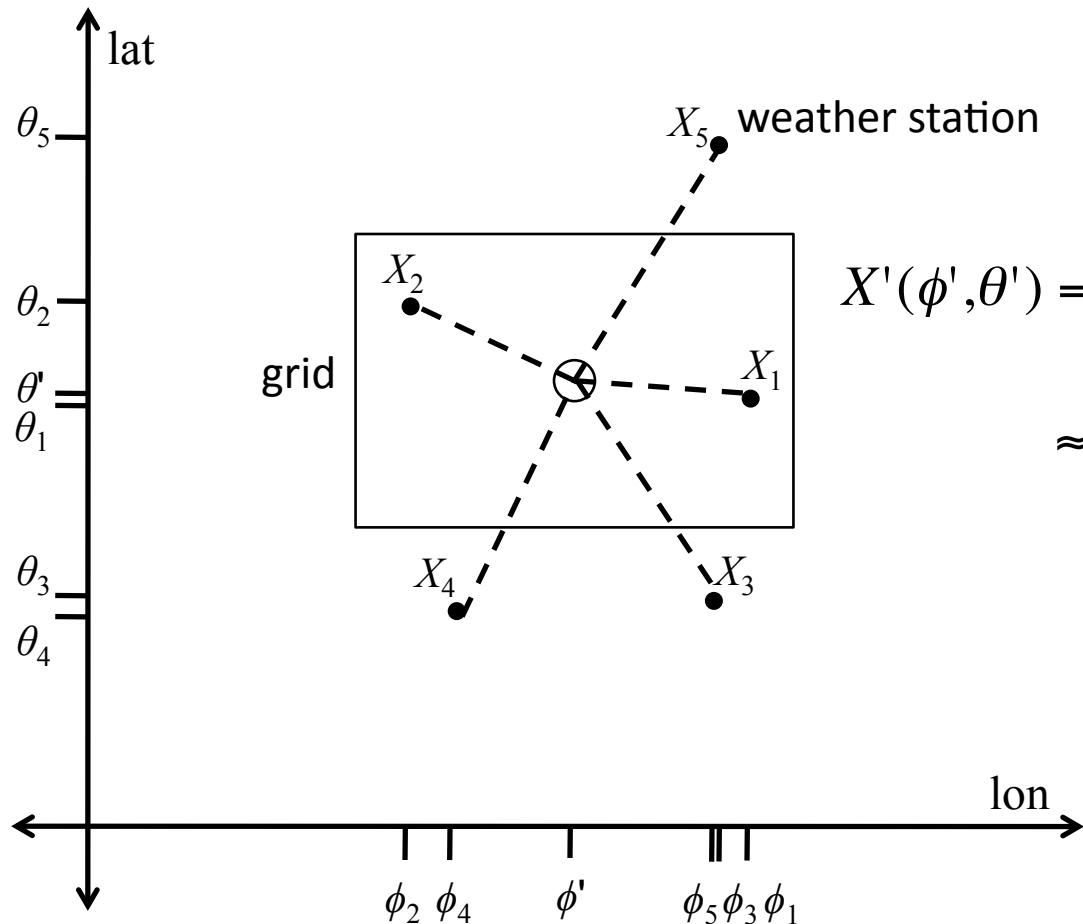


### Key Points

- **Moment patterns are smooth at synoptic scale**
- **Anomaly distributions are regionally identically distributed,  
i.e., equal relative exceedance probabilities**  
( $T_{\min}$ ,  $T_{\max}$ ,  $T_{\text{avg}}$  all seasons)



## Spatial Scale Dependence: The Case of Local Area Averages (i.e., spatial interpolation)



$$X'(\phi', \theta') = A^{-1} \iint_{\theta, \phi} X(\theta, \phi) d\theta d\phi \\ \approx \sum_i w_i(d_i, \phi_i, \theta_i) X_i ; \quad \sum_i w_i = 1$$

# Moment Equations for Weighted Sums of (zero mean) Random Variables

$$\hat{\mu}_n = \overline{\left( \sum_i w_i X_i \right)^n}$$

Mean:

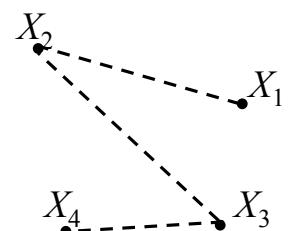
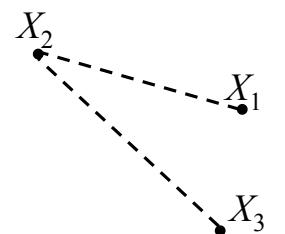
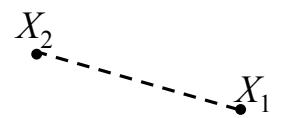
$$\hat{\mu}_1 = \overline{\sum_i w_i X_i} = \sum_i w_i \overline{X_i} = 0$$

Variance:

$$\hat{\mu}_2 = \sum_i w_i^2 \overline{X_i^2} + \sum_{i \neq j} w_i w_j \overline{X_i X_j}$$

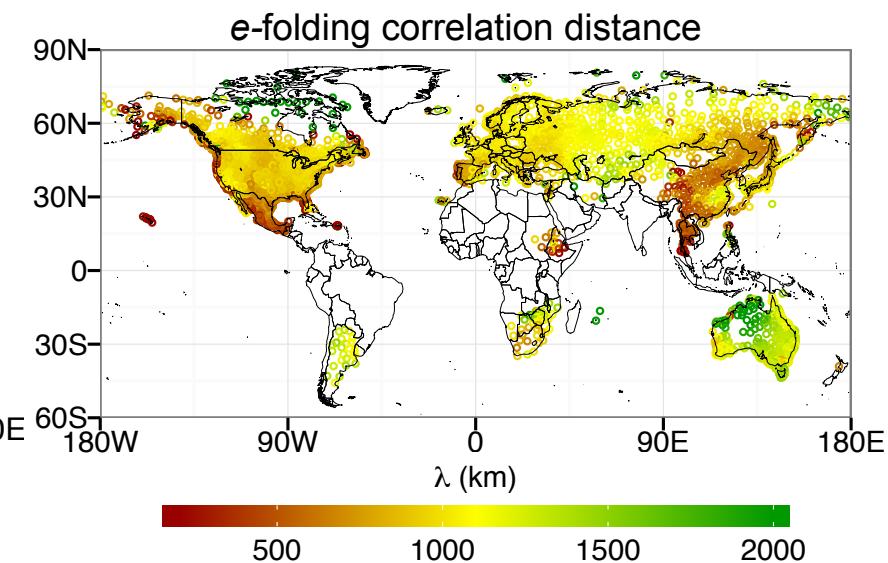
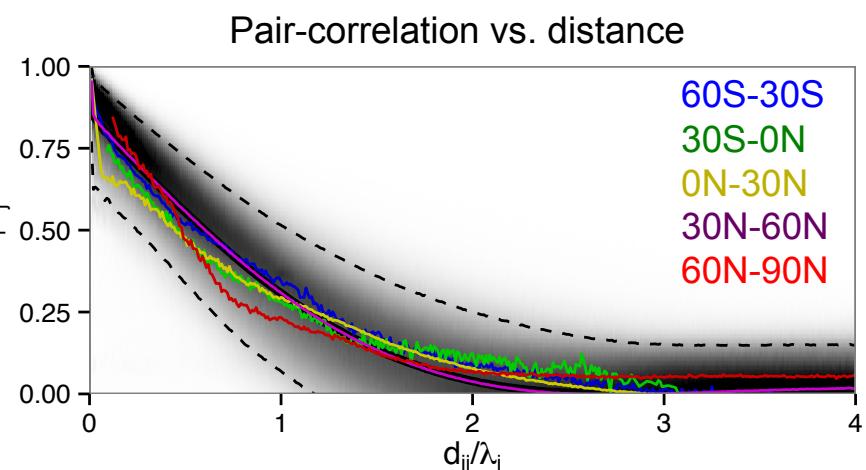
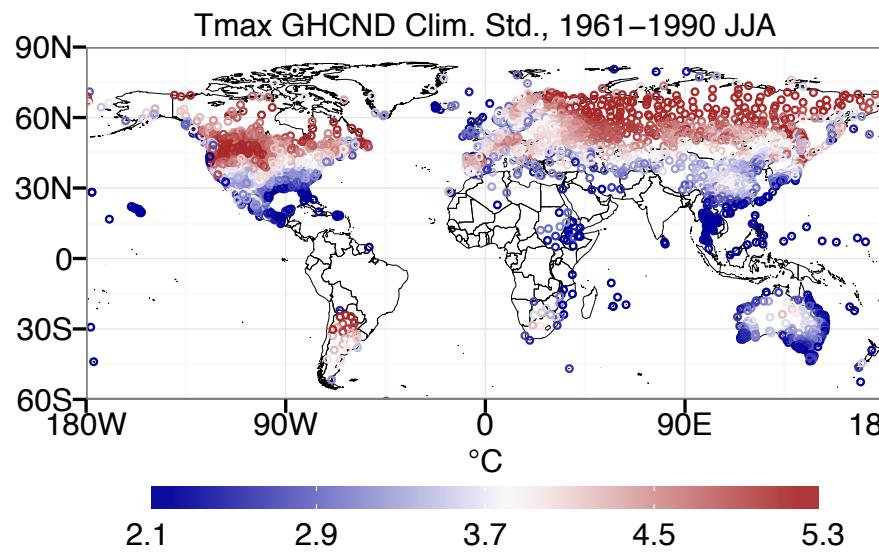
3<sup>rd</sup> Moment:  $\hat{\mu}_3 = \sum_i w_i^3 \overline{X_i^3} + 3 \sum_{i \neq j} w_i^2 w_j \overline{X_i^2 X_j} + \sum_{i \neq j \neq k} w_i w_j w_k \overline{X_i X_j X_k}$

4<sup>th</sup> Moment:  $\hat{\mu}_4 = \sum_i w_i^4 \overline{X_i^4} + 4 \sum_{i \neq j} w_i^3 w_j \overline{X_i^3 X_j} + 3 \sum_{i \neq j} w_i^2 w_j^2 \overline{X_i^2 X_j^2} + \dots$   
 $\dots + 6 \sum_{i \neq j \neq k} w_i^2 w_j w_k \overline{X_i^2 X_j X_k} + \sum_{i \neq j \neq k \neq l} w_i w_j w_k w_l \overline{X_i X_j X_k X_l}$



# Second Order Correlations: A locally homogeneous random field

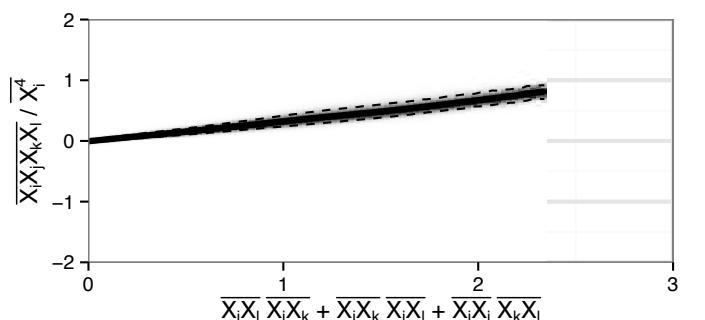
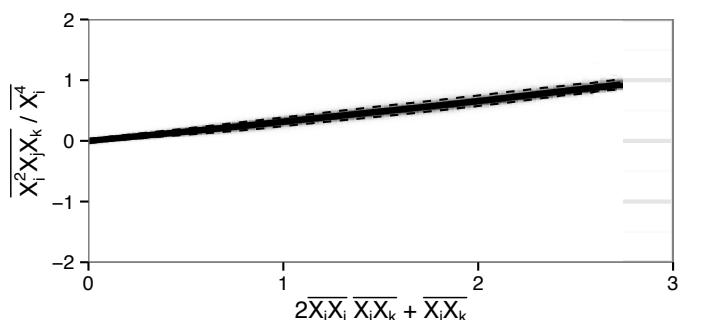
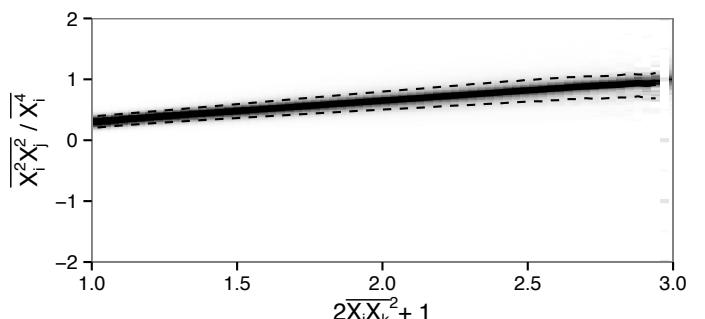
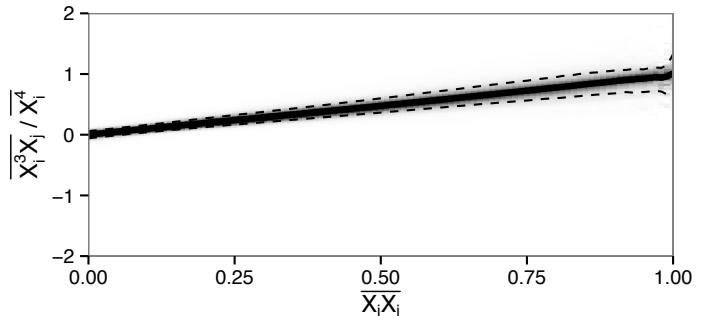
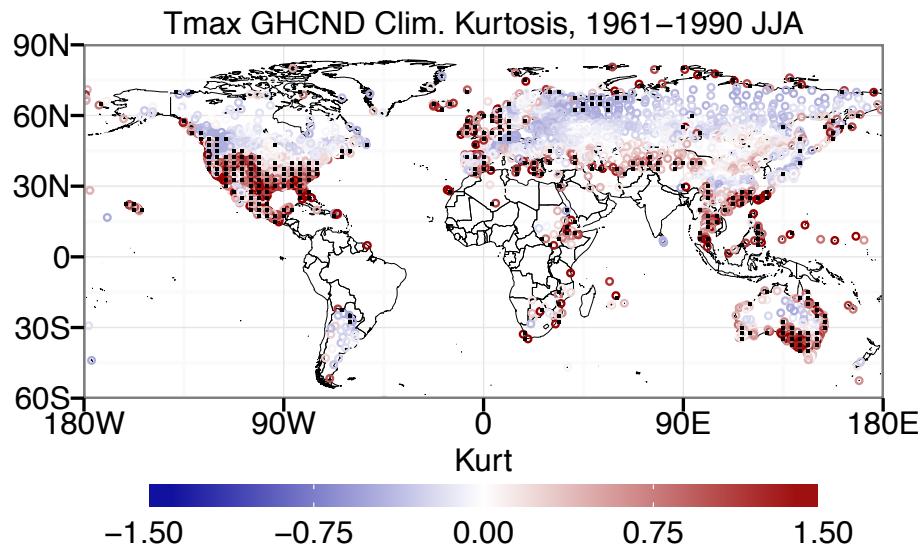
$$\hat{\mu}_2 = \sum_i w_i^2 \overline{X_i^2} + \sum_{i \neq j} w_i w_j \overline{X_i X_j} \longrightarrow |\overline{X}|$$



# Fourth Order Correlations: a Quasi-Gaussian Random Field

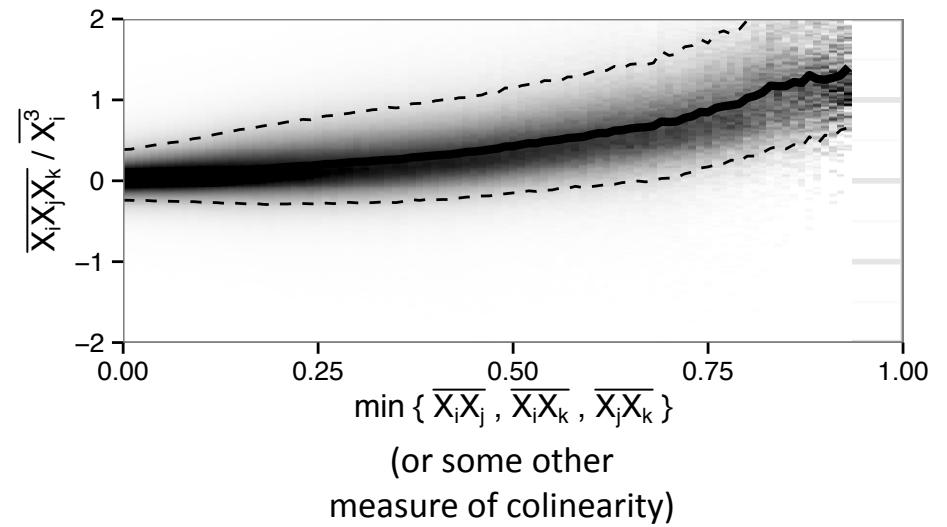
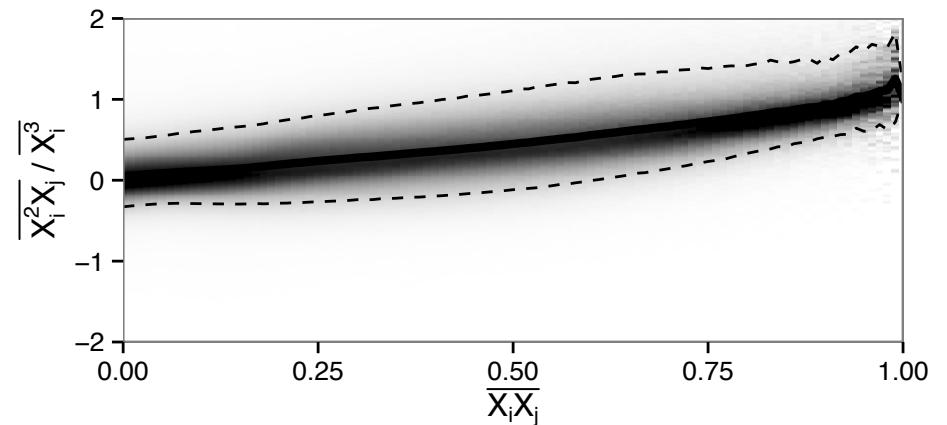
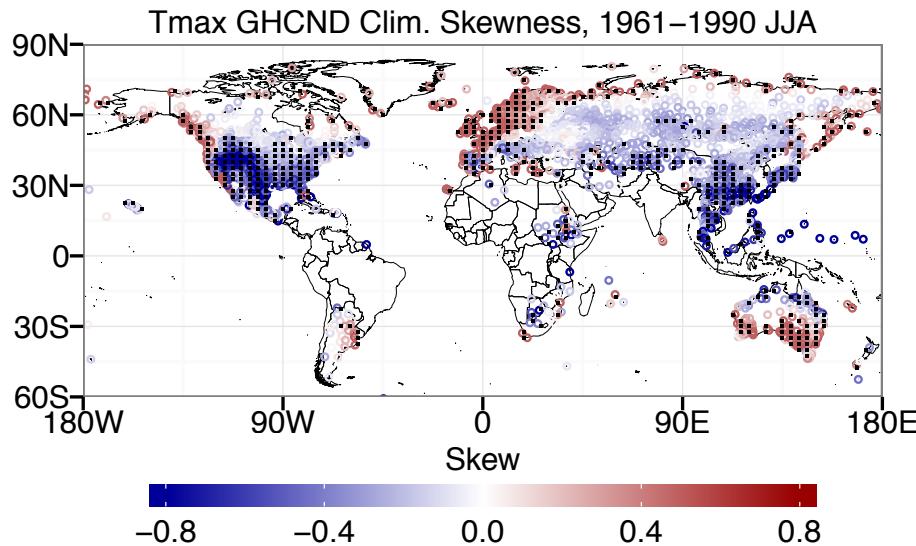
$$\hat{\mu}_4 = \sum_i w_i^4 \overline{X_i^4} + 4 \sum_{i \neq j} w_i^3 w_j \overline{X_i^3 X_j} + 3 \sum_{i \neq j} w_i^2 w_j^2 \overline{X_i^2 X_j^2} + \dots \\ \dots + 6 \sum_{i \neq j \neq k} w_i^2 w_j w_k \overline{X_i^2 X_j X_k} + \sum_{i \neq j \neq k \neq l} w_i w_j w_k w_l \overline{X_i X_j X_k X_l}$$

$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} X_i \quad X_j \quad X_k \quad X_l = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} X_i \quad X_j \quad X_k \quad X_l + \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} X_i \quad X_j \quad X_k \quad X_l + \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} X_i \quad X_j \quad X_k \quad X_l$$

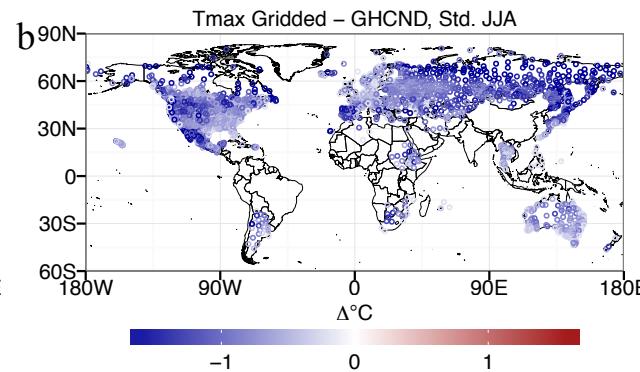
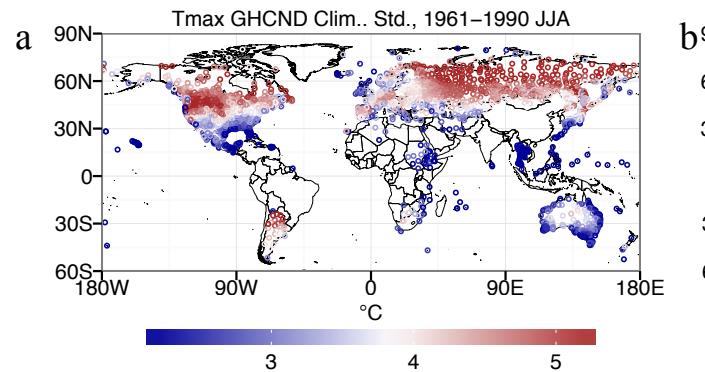


# Third Order Correlations Scale with Colinearity

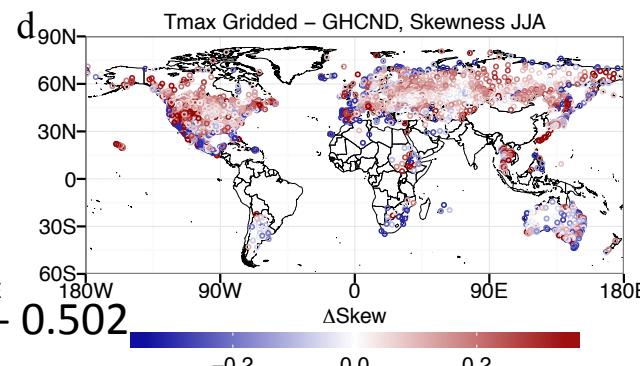
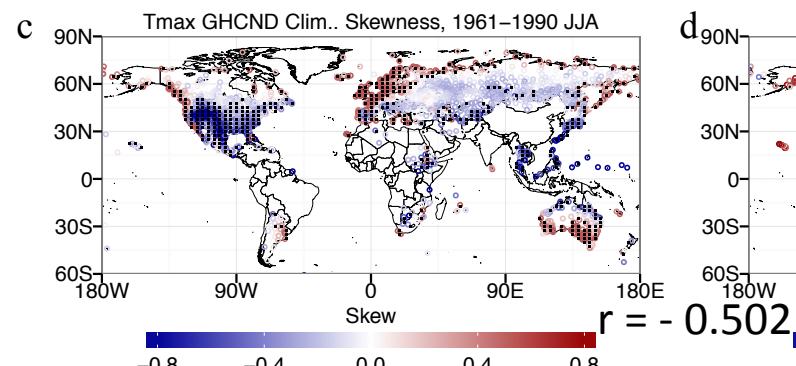
$$\hat{\mu}_3 = \sum_i w_i^3 \overline{X_i^3} + 3 \sum_{i \neq j} w_i^2 w_j \overline{X_i^2 X_j} + \sum_{i \neq j \neq k} w_i w_j w_k \overline{X_i X_j X_k}$$



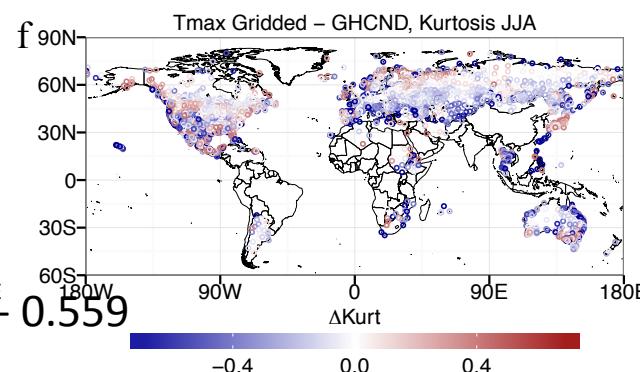
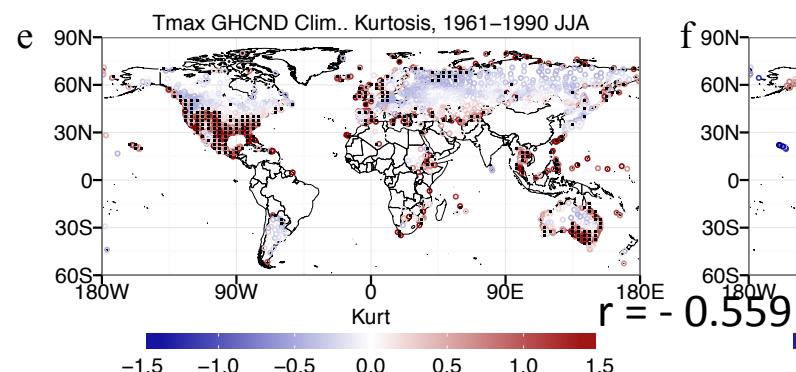
# Moment Climatologies for Interpolated Data, $T_{max}$ JJA



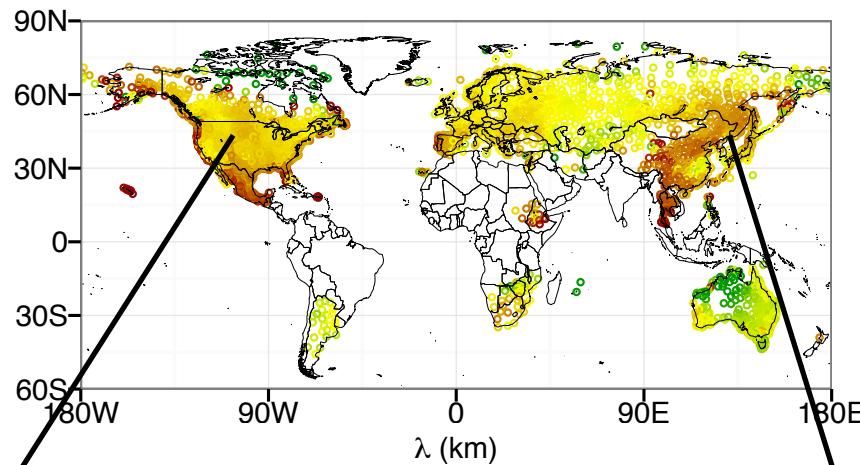
Less variance, globally



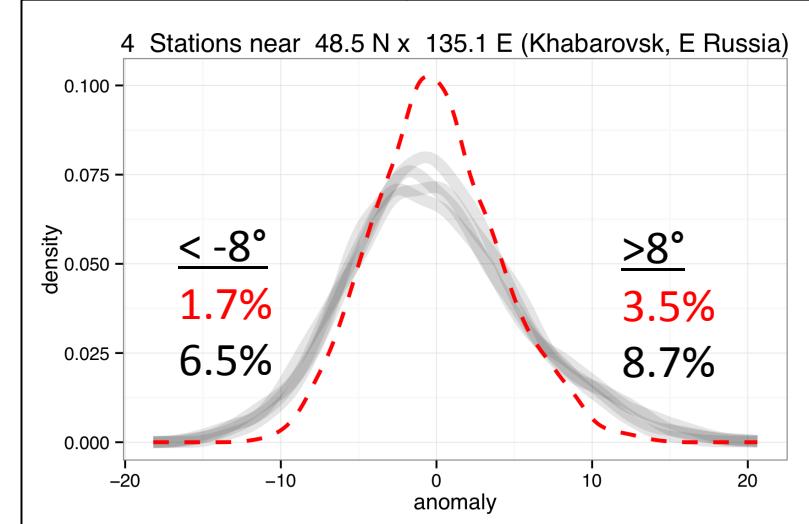
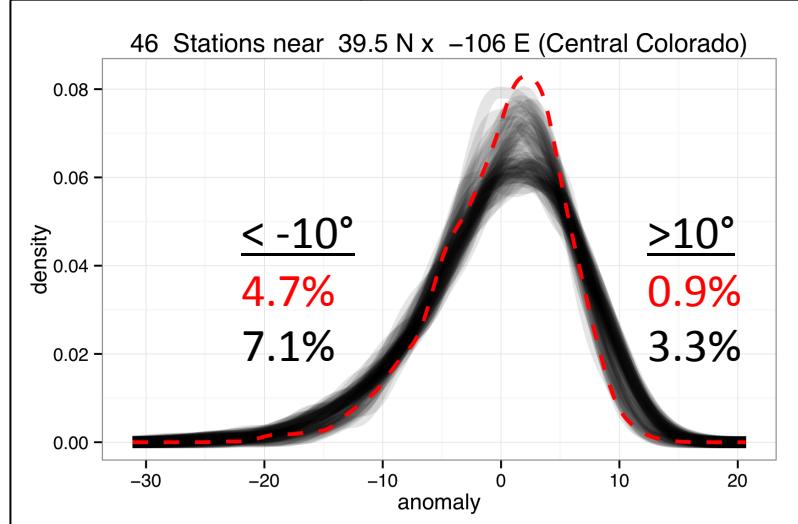
{ More Gaussian,  
Less extreme



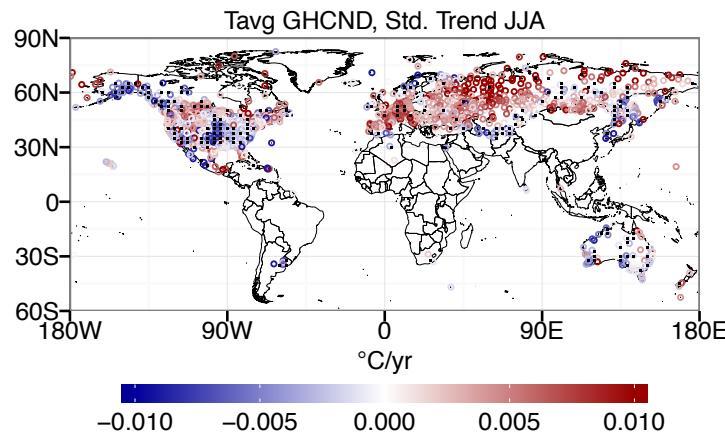
# Point data vs. area averages



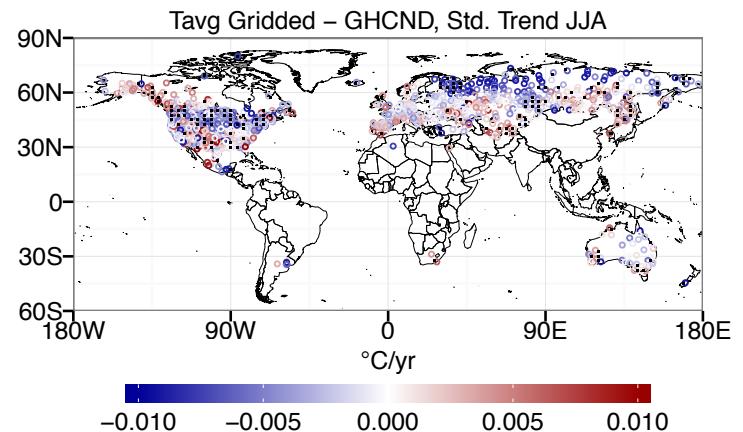
↓ ↓



# Trends in Distribution

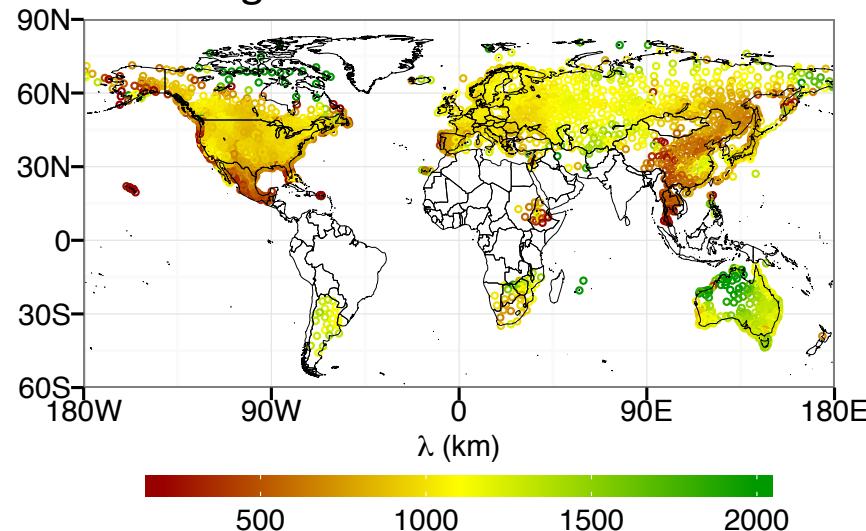


Trends at point-scale  
(Cavanaugh & Shen 2014, J. Clim.)



Trends at grid-scale  
(Submitted to J. Clim.)

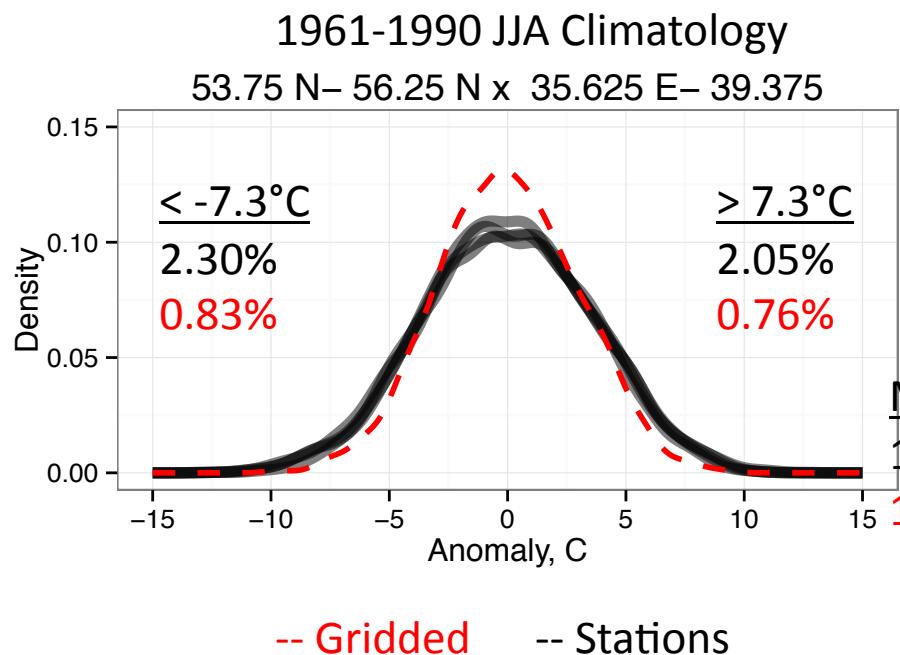
e-folding correlation distance trends?



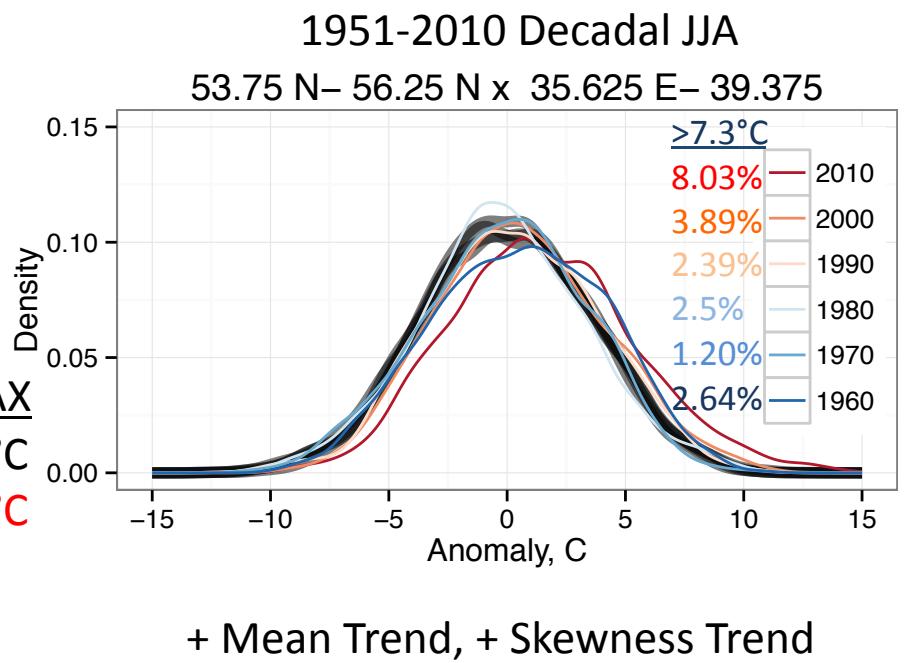
# Putting the Pieces Together

## 2010 Russian Heatwave in PDFs

### Effects of spatial scale



### Trends in distribution



### Summary

- Station variability is identically distributed over large areas
- Gridded data is more normally distributed, less extreme than reality
- The probabilities of daily weather are changing in distribution!
- Trends in distribution can differ across spatial (and temporal) scales!